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INTERDEPENDENCE BETWEEN THE ADVANCE IN SCIENCE AND THE ADVANCE IN TECHNOLOGY

Zenonas Kalinauskas

Bekassinenau 173, D-22173 Hamburg, Germany
E-mail: zenonas@hotmail.de

Abstract. This paper seeks to draw the attention to the fact that the advances in physics are connected with the advances in technology, in particular with the advances in measuring techniques. The mathematical link between physics and measuring techniques is realized using automatic control as the common mathematical core between classical mechanics and the theories of measuring instruments. The reasons for the choice of Cermelo – Fraenkel axiomatic set theory as the most adequate metatheory for the theories of physics and theories of techniques will be given.

Keywords: physics, techniques, automatic control, metatheories, advances.

Introduction

Not very long ago the considerable part of western philosophy was occupied with the problem of advances in natural sciences. This situation has continued up to today and it looks like it will continue in the future. A large part of the research in philosophy of sciences is directed to the problem of advances in physics.

The purpose of this article is to show how the mathematical core of a physical theory (in our case: classical mechanics) is linked to technology: firstly to the theory of automatic control, secondly to that of measuring devices. This link, of course, is meant in logical sense: there exists the common mathematical core between classical mechanics, theory of automatic control and theory of measurement devices. As far as I know this fact was not described in philosophy and history of technology before.

It also shows that historically the advances in physics were dependent of the advances of measuring devices and vice versa: the advances of technology were dependent on the advances of theoretical and experimental physics.

Differing viewpoints about the relationship between physics and technology

An important discovery in modern physics took place at the very beginning of the twentieth century: Classical mechanics has ceased to explain the phenomenon called black body radiation at the frequencies of violet spectrum. In the year 1900 M. Planck

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used the hypothesis that electromagnetic radiation depends on frequency not occurring continuously but at discrete intervals. This was an ad hoc explanation in the area of classical mechanics. Quantum mechanics was discovered by Schroedinger and Heisenberg in the years 1925–1927. It was able to explain many phenomena which classical mechanics could not explain, including the already mentioned black body radiation. It must be stated that as far as black body radiation is concerned, quantum mechanics explained it not ad hoc as in the above mentioned case of Einstein's hypothesis: rather, it followed directly from the axioms of quantum mechanics.

The discovery of quantum mechanics was called later a "scientific revolution" in the history of science. Similar advances in physics were the discoveries of the theories of special relativity and general relativity by Einstein.

Special relativity was able to explain phenomena which classical (non – relativistic) mechanics could not explain. The same statement can be said about general relativity theory.

The transition from classical mechanics to quantum mechanics has meant a significant change in the axiomatic system and main laws of classical mechanics. The same is true for special relativity and general relativity. However, these revolutionary discoveries in physics will not be considered further in this article.

As the title of the article indicates, my purpose is to show the interdependence between the advances in theories of physics and the advances of the theories of technology.

Here I should mention the article of Scheibe (1999: 166), where he shows that he and Krueger differed on the issue of whether the measuring devices belong to the theories of physics or not: "... In the latter group we additionally meet two relations which show that one theory is an auxiliary theory for the other theory. This means having the purpose either to test the other theory or firstly to interpret it empirically. This is necessary, for example, for the purpose to test Newton's sky mechanics. On the other side the theories of how the measuring devices function often simultaneously provide the interpretation of the magnitudes of the theory which must be measured by those devices, e. g. the theory of a galvanometer provides the interpretation of the term "current in a conductor".

As far as physics is concerned I shall first deal with the scope of the advances of the theories of classical mechanics, in particular I shall mention only singular advances from Newtonian Mechanics to Lagrangean and to Hamiltonian mechanics. The main purpose of my article is to describe the application of Hamiltonian mechanics to measuring instrument theory. As already mentioned at the very beginning, I shall discuss the theory of automatic control as the link between Hamiltonian mechanics and measuring instrument theory. More specifically: Hamilton function or "Hamiltonian" has more advantages in the area of applications of classical mechanics than Lagrangean function in Lagrange formulation does, *a fortiori* more advantages than force function in Newtonian formulation. This remark applies only to the use of Hamilton function in classical mechanics.

It is useful to mention that both Hamilton function in classical mechanics and Hamilton operator in quantum mechanics are important to create the intuitive connection between classical mechanics and quantum mechanics.

As far as connection between physics, technology and engineering is concerned I have already mentioned two areas in engineering: theory of automatic control and measurement instrument theory.

Hamiltonian is significant in the theory of automatic control, especially in the area of optimization of automatic control systems.

The significant part of the theories of some measuring devices (at least electric measurement devices) can also be treated as extensions of Hamiltonian mechanics. This is the opinion of Krueger, which is contrary to the opinion of Scheibe as quoted above.

In this connection I would like to mention that both Scheibe and Krueger worked according to the program "Intertheoretic Relations" of the universities of Goettingen and Bielefeld in the seventies of the last century.

Metatheory for of physics and technology

As already mentioned in the introduction, the advanced theories explain more phenomena than their predecessors did. This means that in the area of experimentally accessible sentences the set of correct predictions by the advanced theory is greater than the set predicted by the older theory. To say it the other way: some predictions of the new, advanced theory contradict to the predictions of the old theory. The natural question arises: if the extensions of theories by means of auxiliary theories to the experimentally accessible sentences contradict, do the theories themselves contradict?

Although we wish to concentrate our ideas only to the above mentioned three theories of classical mechanics, let us take momentary because of didactic reasons the following example: classical mechanics – quantum mechanics. We can extend both theories by means of auxiliary theories and reach the protocol sentences of our measurement devices: classical mechanics and quantum mechanics.

Let us take e. g. either Gibbs or Maxwell statistical theories as extensions both of the classical mechanics and of quantum mechanics. They contradict in part of their predictions at the experimental level. Do quantum mechanics, represented by selfadjoint operators in Hilbert space contradict classical (in our case) Hamiltonian mechanics represented by Borel functions in phase space?

The answer of some philosophers of science some decades ago was: the theories can contradict only if they have the same language in their syntactical part, respectively, the same semantical model of their mathematical structures.

The next question is: do the theories of physics and some theories of technology have the same mathematical structure?

In order to find the coincidence or at least correspondence between classical mechanics, automatic control theories and measurement instrument theories we must have a metatheory by means of which we could treat our theories. How should the metatheory look like?

Let us look at our colloquial language.

The explanations of facts of everyday life are presented using words of a colloquial language e. g. English, spoken or written, according to the rules of the English Grammar. The grammar, especially syntax and morphology has its rules but nobody would affirm that the statements of our everyday language are unique. Therefore colloquial languages do not fit for our purposes.

For mathematics, physics and technique we use the rules of the artificial languages which we call logic.

For the purpose of orientation of the following sections we can presuppose the following statement: the mature theories of physics and parts of the mature theories of technology are mathematics, couched in the physical and technical vocabulary.

The weakest logic is propositional logic. Propositional logic is an artificial language: it is study of truth, restricted to the relationship between the truth of one proposition and that of another.

If we can prove in logic everything that is true, and *vice versa* then we can say the logic is complete. It asserts that the existent list of rules (syntactical part of the propositional logic) allows us to deduce every consequence (semantical part of logic). The constants of propositional logic contain also symbols, which are similar to the words of colloquial language: "and", "not", "or", "implies", "if and only if".

Using propositional logic we can prove only the theorems of a very modest part of mathematics: Boolean algebra, "naive" set theory, etc.

First order logic has richer language than propositional logic. It contains not only logical connections between its sentences (as propositional logic does), but additionally the quantifiers for "all" and "there exists".

Before discussing the completeness of first order theories let us speak a while about the concept of "completeness", which is significant both in propositional, first order and in second order logic:

If we can prove everything that is true, then we say that the logic is complete.

If the statement is consistent (not contradictory) in the syntactical part of the theory, then it is valid in the semantical part of the same theory.

Here are the original formulations of Goedel published in the years 1935–1937:

The completeness theorem of first order logic has two forms:

Completeness Theorem, First Form (Goedel):

A formula $\boldsymbol{A}$ of a theory T is theorem of T if and only if is valid in T.

Completeness Theorem, Second Form:

A theory T is consistent if and only if it has a model.

First order logic is richer then propositional logic. Using the above mentioned quantifiers we can axiomatize more mathematical theories and prove uniquely more theorems than we could do using only propositional logic: it can be applied in mathematics also to group theories, algebraic theories, etc. Nevertheless we are using continuous functions in physics and in technology also. In order to define continuous functions we must use the theories of topology.

Using the first order language the function varies in the argument domain over numbers or symbols, of a set. In topology the function can vary in the definition domain over sets and over sets of sets. This is the reason why we need to extend our first order logic so that the objects of our mathematical theory could be able to vary not only over sets but also over sets of sets. The number of sets in topology can be countable infinity. By means of first order language we can not define infinity.

Incompleteness Theorem (Goedel-Rosser). If theory *T* is an axiomatized extension of the countable theory *N*, then *T* is not complete.

(Schoenfield: 132) writes the following commentary to the Incompleteness Theorem:

"The incompleteness theorem has important implications concerning the axiomatic method. The idea of axiomatic method is that, given certain concepts, we introduce a language for expressing facts about these concepts and then introduce an axiom system for proving facts about these concepts. The axiom system must be such that all theorems of the axiom system are true; and we hope that it will be such that all true sentences of the language will be theorems. In any case, we will certainly want the axioms and rules of the axiom system to be such that we can decide what is and what is not a proof. (Otherwise, we could achieve our object by simply adopting all true sentences as axioms.)".

"The incompleteness theorem tells that if *T* is a consistent (not contradictory) axiomatized extension of N, then some closed formula ***A*** of *T* is undecidable in *T*" (Ibid. :133).

We can see that first order logic alone is not adequate to treat mathematics contained in theories of physics and technique for the purpose of their comparison.

Before we pass from the first order logic to the other kinds of logic, respectively to one useful for us, namely Cermelo-Fraenkel set theory, I wish to make an important **remark** in order to prevent possible misunderstandings: mathematics can be treated syntactically as a theory with its extensions and as a structure with its expansions.

Physics and technique can **not** be theories in the sense of mathematical logic. They must be interpreted by theories which are "empirically significant" (the term in quotations was often used some three or four decades ago in the philosophy of science).

According to the textbooks of mathematical logic, the interpreted theory is a structure and not a theory. Instead of speaking about expansions of structure in the sense of mathematical logic, we can also speak about applications in case of a general theory of physics or of a general theory of technology.

Let us return to Incompleteness Theorem: In order to immunize **physical** theories on the metatheoretical level and make them immune against the possible reproaches from the direction of Goedel's Incompleteness theorem.

One possible way to master this situation would be to choose first order many sorted logic. In case of topology we could choose one sort of variables for sets the other sort of variables for sets of sets and so on. This would be a very complicated procedure.

Ludwig has proposed in the years between 1970 and 1978 Cermelo – Fraenkel axiomatic set theory. Ludwig and Thurler (2006) have made some inessential changes in respect to the previous editions of Ludwig. Additionally some easily solvable exercises and illustrations were added.

The reasons to use Cermelo – Fraenkel CF (C) set theory for logical reconstruction of **physical** theories can be the following: already in the first version of the year 1908 the German word "Urelement" was used. The concept "Urelement" became internationally used. In Cermelo – Fraenkel (C) set theory the word Urelement is used to denote **both** mathematical and empirical concepts and physical things.

Cermelo – Fraenkel (C) set theory is first order logic, extended by means of explicit definitions with the binary predicate symbol $x \in y$. "We intend that the individuals of ZF shall be pure sets, and that shall mean that x is a member of y" (Schoenfield: 239–245).

We can reconstruct the theories of physics and technology by means of axiomatic set theory.

Bourbaki (1968) proposed to reconstruct the theories of mathematics dividing the structures of mathematics into three parts: main base sets, auxiliary base sets and typified sets.

He proposed also the "procedure of deduction of the poorer species of structures from the richer species of structures".

One of his simple examples is: the deduction of topological species of structures and group species of structures from the richer species of structures, namely from the structure of topological groups.

Advance in classical mechanics: Newtonian, Lagrange, Hamilton

Newtonian mechanics was published in *Principia Mathematica* in 1687. According to its second law, it described its objects in terms of forces and accelerations. The description was suitable using Cartesian coordinate system. Lagrange published his Mecanique Analytique in 1788. It described its objects in generalized coordinates. Therefore it was possible to solve more problems using Lagrangean formulation, eliminating the forces of constraint. Nevertheless it had the disadvantages when applied to dissipative systems.

Hamiltonian mechanics was completed in 1834. It is more flexible in its choice of coordinates for solving problems. Lagrangean formalism centers on Lagrangean function L.

If we denote the kinetic energy T, potential energy U, then for most systems of interest, L is just the difference of the kinetic and potential energies:

$L = T - U.$

The 2n coordinates define a point in state space and specify a set of initial conditions (at any chosen time t_0) that determine a unique solution of n second-order differential equations of motion, Lagrange equations.

The Hamiltonian approach leads to Hamilton equations for a system with n degrees of freedom, the Hamiltonian approach gives $2n$ first-order differential eqations , instead of the n second-order equations of Lagrange.

H ($\boldsymbol{q}$, $\boldsymbol{p}$, t) may have an explicit time dependence, as indicated by the final argument t, and this also makes H vary with time. Mathematically, this means that the derivative of Hamiltonian contains $2n+1$ terms.

Today in classical Hamiltonian mechanics usually $2n$-dimensional cotangent bundle of the n-dimensional configuration space is chosen.

The above three formulations of classical mechanics (Newtonian, Lagrangean and Hamiltonian) are equivalent in the sense of logic. This means that one theory can be deduced from the other and *vice versa* by means of formal logic. On the other hand this does not mean that the applications of the Hamiltonian formulation are equivalent to the applications in formulation of Lagrangean mechanics.

Concrete example: the important Liouville's theorem:

Two identical systems that start out with nearly identical initial conditions move rapidly apart in phase space and if the motion is chaotic, then at least some pairs of points inside the volume must move rapidly apart. But the total volume cannot change. Therefore as the volume grows in one direction, it must contract in another direction, becoming like a cigar.

The proof of Liouville's theorem depends only on the validity of Hamilton's equations and there is no equivalent proof of Liouville's theorem Lagrangean theory.

Hamiltonian formalism uses Hamilton function H which denotes the total energy.

H has clear physical significance and is frequently conserved not only in physics but also in automatic control theory.

Quantum mechanics as theory of microphysics was mentioned in the introduction. It uses Hamilton operator to define total energy of a quantum mechanical object. In order to visualize transition of the objects of classical mechanics to the quantum mechanical objects the so called "quantization rules" were introduced in the past. They have some significance also today.

As far as automatic control theory is concerned we can tell in advance that the recent automatic control theory and the measurement instrument theory have no direct application to quantum mechanics: all applications of quantum mechanics can be measured only by classical measurement instruments.

We have seen that as far as the area of validity of classical mechanics is concerned Hamiltonian mechanics has many advantages. One of the applications of classical mechanics is the theory of automatic control theory. Automatic control theory includes optimization problem. The maximum principle of academician Pontryagin can be applied to solve the tasks of optimal control. The input there is represented by a discrete, not continuous time function Dambrauskas (2003: 154).

For the engineers who wish to specialize for automatic control the simplest problem of obtaining the optimal control of a time invariant and continuous dynamical system was proposed (Doolin 1990: 116). Often specializing in automatic control is started using Riccati equation which presupposes the Hamiltonian classical mechanics. The only difference from physics in this case is that we use Hamilton matrix, as the algebraic extension of the Hamilton function.

Until recent we treated Hamiltonian mechanics only in so-called time domain. Sometimes it is more convenient to treat the objects of the automatic control theory and the theory of measuring devices in frequency domain.

Using either Fourier or Laplace transform or, as an alternative: Laplace transform and inverse Laplace transform we can get transfer function.

The transfer function is defined as the quotient of Laplace transform of the output vector to the input vector.

Hamiltonian mechanics, automatic control, measurement devices

An ideal measuring device must reproduce flawlessly the output of the temporal process of the measured magnitude (Kiencke 2008: 85). Let is mention first for example the design of a manometer. It can have a nonlinear behavior. In order to describe the **dynamic** behavior of the measurement device a step signal is used as input. The deviation of the step response from the ideal step response is considered as a mistake.

As far as the evaluation of step response is concerned the difference between periodic and non periodic adjustment must be taken into account.

There is the other method for the determination of the dynamic behavior of the frequency response, namely the method described in the previous section:

The determining of the design of measuring device in the frequency domain, is usually used in automatic control theory. As already pointed out in the previous section the frequency domain is equivalent to time domain. The treatment in time domain is one of applications of Hamiltonian mechanics. The equivalence can be proved by means of Laplace transformations in both directions.

Environment for measurement devices

Since there are no universal theories of measurement, Piotrowski (1992: V–VI) proposes four axioms which, according to his words, have been known and used for some time: Isomorphic relations occur between the states of a given quantity and the values of that quantity.

The mapping of a state of a given quality into an image of the state is ambiguous – a point is mapped into a set.

The ambiguity of the mapping of a state into an image, produced with a measuring device, may be determined from the mathematical model which describes the metrological properties of the instrument.

The declared image of reality is related to some agreed reference states.

Ludwig (2006: 85) proposes **species of uniform structures** which maybe can describe the physical and technical reality more adequately. For the purpose of measurement technique it is better to describe the mathematical language of physical and technical reality theory not using the structures of (topological) Euclidean metric but uniform structures.

If we have registered the finite set of measured magnitudes and if we have decided to take the value **a**, then using uniform structures: "……… has the advantage, that the definition of the **neighbourhood of a** and at the same time the values which in the framework of measurement accuracy can not be distinguished from **a** do not depend on **a**" (Scheibe 1997: 97).

Therefore if we are measuring the mass of a planet the difference in 1 gram does make any meaning. On the other side if we are measuring mass in the microscopic area with the accuracy 1 gram, the measurement makes no sense. The conclusion: we get the amendment of the situation if we use not absolute but relative mistakes of our measurements and uniform metric structures for the evaluation of the results of measurements instead topological metric structures. We have already noticed that uniform spaces are generalizations of metric spaces.

Conclusions

The recent literature in the philosophy of physics and technology describes various aspects both of technology and of physics. It is difficult to find enough literature, which shows the interdependence between the mathematical cores of physics and technology referred to important inventions and discoveries.

Mathematical logic and axiomatic set theory used to systematize progress in physics were already mentioned in this article. This article shows that the same metatheories can be used to discuss the interdependence between physics and technology.

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ABIPUSĖ PRIKLAUSOMYBĖ TARP PAŽANGOS MOKSLE IR PAŽANGOS TECHNOLOGIJOJE

Zenonas Kalinauskas

Bekassinenau 173, D-22159 Hamburg, Vokietija
El. paštas zenonas@hotmail.de

Santrauka. Dabartinėje literatūroje, skirtoje pažangos technikoje ir pažangos technologijoje aprašymui, abiejų žmonijos veiklos sričių pasiekimai svarstomi nekreipiant dėmesio į abipusę priklausomybę. Straipsnio tikslas – paminėti, kad abiejų sričių matematinės struktūros turi bendrą branduolį. Bendras branduolys šiandien aprašomas tik aptariant fizikos teorijų pažangą. Nurodoma, kad tarp Hamiltono mechanikos, automatinio valdymo kontrolės ir matavimo prietaisų egzistuoja bendras matematinių struktūrų branduolys.

Reikšminiai žodžiai: fizika, technika, automatinis valdymas, metateorija, pažanga.

Zenonas KALINAUSKAS. Baigė Leningrado admirolo S. Makarovo aukštąją jūrų inžinierių mokyklą. Moksliniai interesai: fizikos ir technologijos tarpusavio ryšys, matematinė logika.

Zenonas KALINAUSKAS. Graduate of the Leningrad High Marine Engineering School after admiral Makarov. Research interests: interconnections between physics, technology, mathematical logic.